Mechatronic Control Systems:

An Implementation Perspective

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Outline

- Mechatronic Control Problems
- Discrete Control Systems
- Continuous Control Systems
  - Design Methodologies
  - Aliasing (Covered Thursday 27 Oct)
  - Arithmetics (Covered Thursday 27 Oct)
  - Computation Delay
  - Implementation Paradigms
- Example: PID Controller

More Information

The material presented in these two lectures is primarily based on the following courses:

- Reglerteknik AK (Basic Course in Automatic Control)
- Real-Time Systems
- Nonlinear Control and Servo Systems

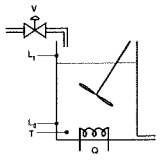
More Information cont.

Strong Relations to other courses:

- Automation
- Embedded Systems
- Multivariable Control (Flervariabel Reglering)
- Real-Time Programming
- Electronics (Elektronik) - analog computations, OP-amplifiers
- Design of Digital Circuits (Digitalteknik) - finite state machines, boolean functions
- Datorteknik - arithmetics
- Numerical Analysis
- Computational Mechatronics
- and many more ...

A Typical Control Problem

Raw material buffer tank with heating (non-mechatronic, but still)



Goals:

- Temperature control: PI-controller
- Level control: open V when level below L_0, keep the valve open until level above L_1
- Sensor fault detection: Generate alarm whenever L_1 is true and L_0 is false

Characteristics

- Concurrent activities.
- Timing requirements – more or less hard.
- Discrete (binary) and analog signals.
- Continuous (time-driven) control and Discrete (event-driven) control
- Discrete control consists of both sequence control logic (state-machine oriented) and combinatorial logic (interlocks) (Alarm = L_1 AND NOT L_0)

The above characteristics hold for almost all control applications, whether mechatronic or not.

Mechatronic System Characteristics

Mechatronic systems are often embedded systems.

The "computing device" is an embedded part of a mechatronic device/equipment.

Constraints on cost and size generate constraints on execution time, "execution space" (memory, word-length, chip-size), power usage, bandwidth, fault-tolerance, ...

"Resource-Constrained Control"

Implementation Technologies

- Analog Computations
  - Pure mechanics
  - Pneumatics
  - Discrete Analog Electronics
  - Analog ASICs
- Digital Computations
  - Electro-mechanical relay systems
  - Discrete Digital Electronics
  - Digital ASICs
  - PLA (Programmable Logic Arrays)
  - FPGA (Field-Programmable Gate Arrays)
  - Digital signal processors (DSP)
  - General Computers (micro-controllers, micro-processors, ...)

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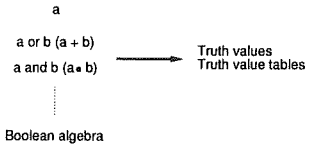
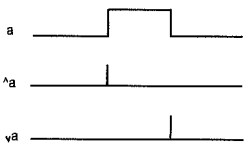
Basic Elements

- Boolean (binary) signals – $0, 1$, $false, true$, $a, \bar{a}$
- expressions



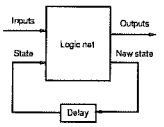
- events



Logic Nets

- Combinatorial nets - boolean functions
  - outputs = f(inputs)
  - interlocks, "förreglingar"
- Sequence nets
  - newstate = f(state,inputs)
  - outputs = g(state,inputs)
  - state machines (automata)

Asynchronous nets or synchronous (clocked) nets



Course on Design of Digital Circuits

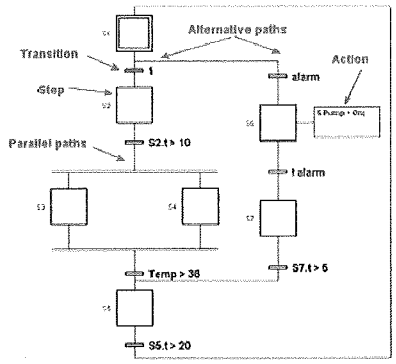
Grafcet

Extended state machine formalism for implementation of sequence control

Industrial name: Sequential Function Charts (SFC)

Defined in France in 1977 as a formal specification and realization method for logical controllers

Standardized in IEC 848. Part of IEC 1131-3



Editors and compilers.

Statecharts

D. Harel, 1987

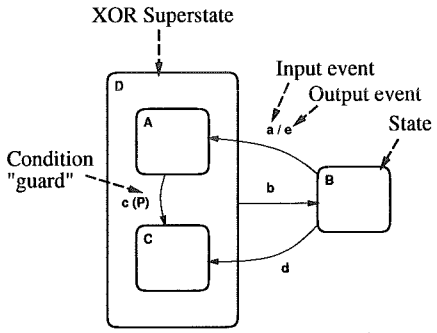
Statecharts =

- state-transition graphs
- hierarchy
- concurrency
- history

The state-machine formalism used within UML (Unified Modeling Language).

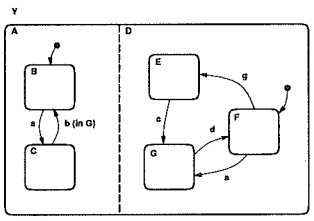
Several different tools available.

Statechart Syntax



Statecharts Concurrency

AND Superstates:



Y is the *orthogonal product* of A and D

When in state (B,F) and event **a** occurs, the system transfers *simultaneously* to (C,G).

Implementation alternatives

Several possibilities:

- discrete digital electronics
- digital ASICs
- PLA (AND-gates and OR-gates, minimal conjunctive form)
- FPGA (more general gates)
- Processors
  - single-bit CPUs (simple PLCs (Programmable Logic Controllers))
  - micro controllers/processors

What to choose depends on cost, size, requirements on flexibility, ...

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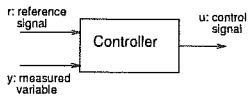
Continuus Control Systems



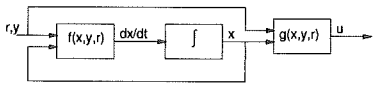
General Controller Form:

$$\begin{aligned} \frac{dx}{dt} &= f(x,y,r) \\ u &= g(x,y,r) \end{aligned}$$

Linear case:

$$\begin{aligned} f(x,y,r) &= Fx + Gy + Hr \\ g(x,y,r) &= Cx + Dy + Er \end{aligned}$$

Implementation



Integration + function generation

Linear case:

- summation + accumulation
- multiplication with coefficient
- scalar product

Non-linear elements:

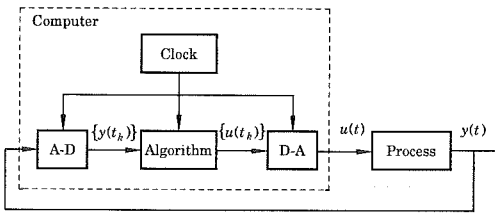
- selector logic (min/max, comparison)
- general non-linear elements for reference signal generation, gain-schedules, adaptation (can often be implemented as lookup-tables)

Discrete-time Implementation

Digital controllers can be designed in two different ways:

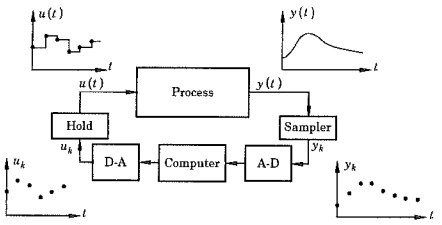
- Discrete time design
  - sampled (digital) control theory
  - shift operators (z-transforms)
  - $u(k) = k_1 y(k) + k_2 u(k-1)$
  - h a design parameter
- Continuous time design + discretization
  - Laplace transform
  - $U(s) = G_c(s)E(s)$
  - approximate the continuous design
  - fast, fixed sampling

Sampled Control Theory



The basic idea: Look at the sampling instances only!

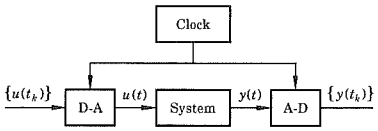
Sampled Control Theory



- System theory analogous to continuous time linear systems
- Better performance can be achieved
- Problems with inter-sample behavior

Sampling of Systems

Look at the system from the point of view of the computer



Zero-order-hold sampling of a system

- Let the inputs be piecewise constant
- Look at the sampling points only
- Use linearity and calculate step responses when solving the system equation

Sampling a continuous-time system

System description

$$\begin{aligned} \frac{dx}{dt} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

Solve the system equation

$$\begin{aligned} x(t) &= e^{A(t-t_k)}x(t_k) + \int_{t_k}^{t} e^{A(t-s')}Bu(s')\,ds' \\ &= e^{A(t-t_k)}x(t_k) + \int_{t_k}^{t} e^{A(t-s')}\,ds'\,Bu(t_k) \quad (u \text{ const.}) \\ &= e^{A(t-t_k)}x(t_k) + \int_{0}^{t-t_k} e^{As}\,ds\,Bu(t_k) \quad \text{(variable change)} \\ &= \Phi(t,t_k)x(t_k) + \Gamma(t,t_k)u(t_k) \end{aligned}$$

The General Case

$$\begin{aligned} x(t_{k+1}) &= \Phi(t_{k+1},t_k)x(t_k) + \Gamma(t_{k+1},t_k)u(t_k) \\ y(t_k) &= Cx(t_k) + Du(t_k) \end{aligned}$$

where

$$\begin{aligned} \Phi(t_{k+1},t_k) &= e^{A(t_{k+1}-t_k)} \\ \Gamma(t_{k+1},t_k) &= \int_0^{t_{k+1}-t_k} e^{As}ds\ B \end{aligned}$$

Periodic sampling

Assume periodic sampling, i.e. $t_k = k \cdot h$, then

$$\begin{aligned} x(kh+h) &= \Phi x(kh) + \Gamma u(kh) \\ y(kh) &= Cx(kh) + Du(kh) \end{aligned}$$

where

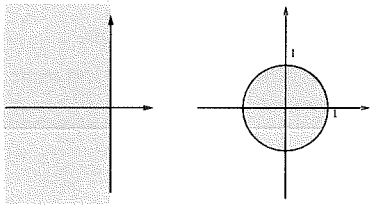
$$\begin{aligned} \Phi &= e^{Ah} \\ \Gamma &= \int_0^h e^{As}\,ds\,B \end{aligned}$$

NOTE: Time-invariant linear system!

Stability region

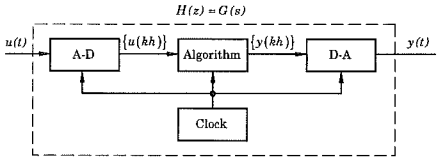
In continuous time the stability region is the complex left half plane, i.e,, the system is stable if all the poles are in the left half plane.

In discrete time the stability region is the unit circle.



Discretization of Continuous Time Design

Basic ideas: Reuse the design



$G(s)$ is designed based on analog techniques

Want to get:

- A/D + Algorithm + D/A $\approx G(s)$

Methods:

- Approximate s, i.e., $G(s) \Rightarrow H(z)$
- Other methods

Approximation Methods

Forward Difference (Euler forward method)

$$\frac{dx(t)}{dt} \approx \frac{x(t+h)-x(t)}{h}$$

$$s = \frac{z-1}{h}$$

Backward Difference (Euler backward method)

$$\frac{dx(t)}{dt} \approx \frac{x(t)-x(t-h)}{h}$$

$$s = \frac{z-1}{zh}$$

Approximation Methods, cont

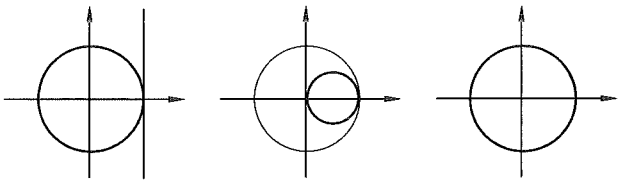
Tustin (trapeziodal, bilinear):

$$\frac{\dot{x}(t+h)+\dot{x}(t)}{2} \approx \frac{x(t+h)-x(t)}{h}$$

$$s = \frac{2}{h}\,\frac{z-1}{z+1}$$

Stability of Approximations

How is the continuous-time stability region (left half plane) mapped?



Forward differences | Backward differences | Tustin

Example: Discretization

Assume that the following simple controller (filter) has been designed in continuous-time:

$$U(s) = \frac{1}{s+2}E(s)$$

Discretize this controller using Forward Euler approximation, i.e. replace s with $\frac{z-1}{h}$:

$$U(z) = \frac{1}{\frac{z-1}{h}+2}E(z)$$

$$U(z) = \frac{h}{z-1+2h}E(z)$$

$$(z-1+2h)U(z) = hE(z)$$

$$u(k+1)-(1-2h)u(k) = he(k)$$

$$u(k) = (1-2h)u(k-1)+he(k-1)$$

Alternative: Write as differential equation first:

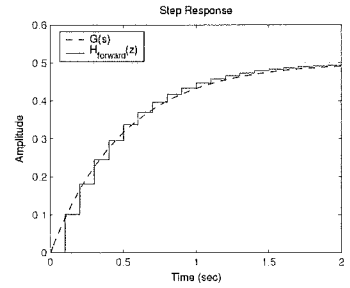
$$\frac{du}{dt} + 2u(t) = e(t)$$

$$\frac{u(k+1)-u(k)}{h} + 2u(k) = e(k)$$

$$u(k+1)-u(k)+2hu(k) = he(k)$$

$$u(k) = (1-2h)u(k-1)+he(k-1)$$

Simulation ($h = 0.1$):



Basic Operations

Integration ⇒ Summation + accumulation

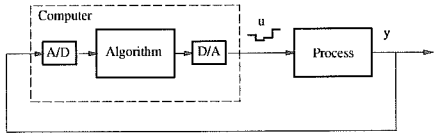
Derivation ⇒ Difference approximation (in the simplest case)

Selector logic and nonlinearities straightforward

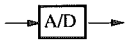
Scalar products main operation in controllers and filters

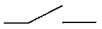
DSPs optimized for this.

Issues: Sampling



AD-converter acts as sampler





DA-converter acts as a hold device

Normally, zero-order-hold is used ⇒ piecewise constant control signals

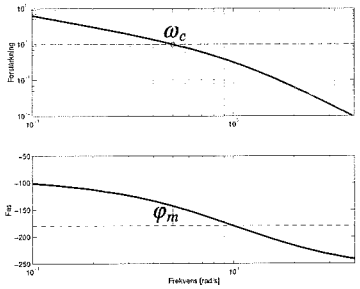
Choice of sampling interval

Nyquist's sampling theorem:

> "We must sample at least twice as fast as the highest frequency we are interested in"

- What frequencies are we interested in?

Typical loop transfer function $L(i\omega) = P(i\omega)C(i\omega)$:



- ω_c = cross-over frequency, φ_m = phase margin
- We should have $\omega_s \gg 2\omega_c$

Sampling interval rule of thumb

A sample-and-hold (S&H) circuit can be approximated by a delay of $h/2$.

$$G_{S\&H}(s) \approx e^{-sh/2}$$

This will decrease the phase margin by

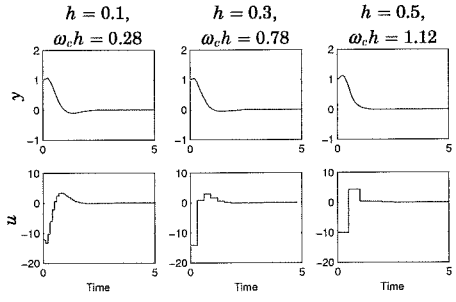
$$\arg G_{S\&H}(i\omega_c) = \arg e^{-i\omega_c h/2} = -\omega_c h/2$$

Assume we can accept a phase loss between 5° and 15°. Then

$$\boxed{0.15 < \omega_c h < 0.5}$$

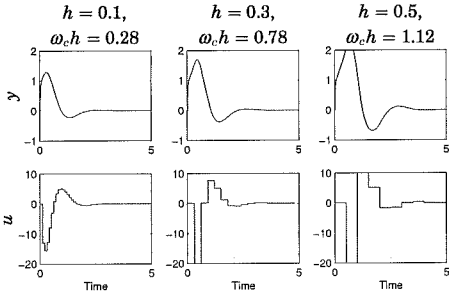
This corresponds to a Nyquist frequency about 6 to 20 times larger than the crossover frequency

Example: control of inverted pendulum



- Large $\omega_c h$ may seem OK, but beware!
  - Digital design assuming perfect model
  - Controller perfectly synchronized with initial disturbance

Pendulum with non-synchronized disturbance



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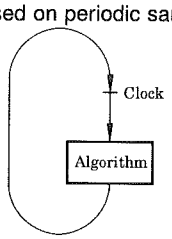
Issues: Computational Delay

Most controller are based on periodic sampling.



Basic problem: $u(k) = f(y(k), .)$

Computation time not accounted for.

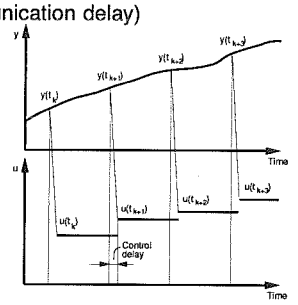
Computational Delay

Problem: $u(k)$ cannot be generated instantaneously at time k when $y(k)$ is sampled

Delay (computational delay or input-ouput latency) due to computation time (and communication delay)

```
LOOP
  wait for clock interrupt;
  read analog input;
  perform calculations;
  set analog output;
END;
```

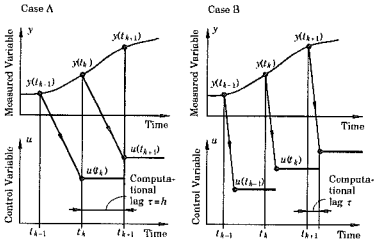
Control delay



Four Approaches

1. Design the controller to be robust against variations in the computational delay
   - complicated
2. Ignore the computational delay
   - often justified, since it is small compared to h
   - write the code so that the delay is minimized, i.e., minimize the operations performed between AD and DA
   - divide the code into two parts: CalculateOutput and UpdateStates
3. Compensate for the computational delay
   - include the computational delay in model and the design
   - sampling of systems with time delays
   - write the code so that the delay is constant

4. Include an delay of one sample in the controller
   - do not send out the control signal until the start of next sample
   - computational delay $= h$
   - easier way to compensate (multiple of the sampling interval)



Minimize Control Delays

General Controller representation:

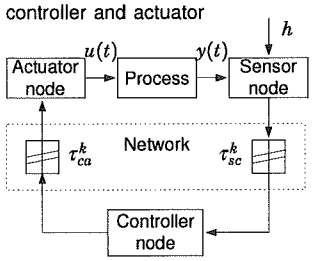
$$\begin{aligned} x(k+1) &= Fx(k) + Gy(k) + G_r y_{ref}(k) \\ u(k) &= Cx(k) + Dy(k) + D_r y_{ref}(k) \end{aligned}$$

As little as possible between AdIn and DaOut

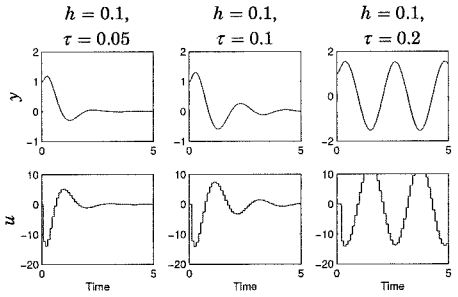
```
PROCEDURE Regulate;
BEGIN
  AdIn(y);
  (* CalculateOutput *)
  u := u1 + D*y + Dr*yref;
  DaOut(u);
  (* UpdateStates *)
  x := F*x + G*y + Gr*yref;
  u1 := C*x;
END Regulate;
```

Other sources of time delays

- Deadtime in the process
  - deadtime after the actuator
  - deadtime before the sensor
- Communication delays
  - between sensor and controller
  - between controller and actuator



Pendulum controller with time delay



- No delay compensation

Delay margin

Suppose the loop transfer function without delay has

- cross-over frequency ω_c
- phase margin φ_m

Phase margin loss due to delay:

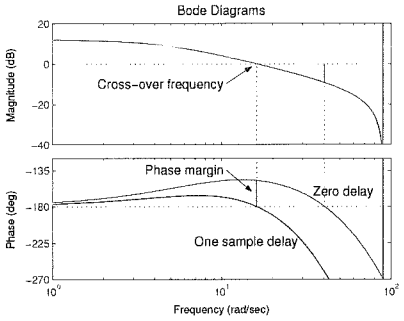
$$\arg e^{-i\omega_c\tau} = -\omega_c\tau$$

Closed-loop system stable if

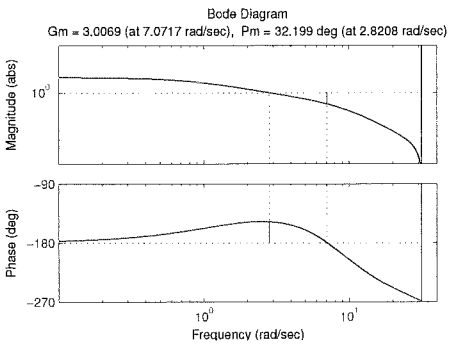
$$\omega_c\tau < \varphi_m \quad \Leftrightarrow \quad \tau < \frac{\varphi_m}{\omega_c}$$

$\tau_m = \dfrac{\varphi_m}{\omega_c}$ is called the **delay margin**

Why is delay bad?



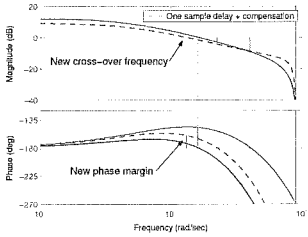
Example: delay margin for pendulum controller



$\varphi_m = 32°$, $\omega_c = 2.8$ rad/s $\Rightarrow$ $\tau_m = \frac{32\pi}{180 \cdot 2.8} = 0.2$

Delay Compensation

If the delay is constant and known, it is possible to compensate for it in the design.



Continuous-time: Smith predictor or lead compensation

Discrete-time: Include the delay in the process model

Why is jitter bad?

The effects of sampling jitter and input-output latency jitter are quite hard to analyze.

If one can measure the actual jitter every sample, it is possible to design controllers that, at least partly, can compensate for the jitter.

For sampling jitter, this corresponds to re-sample the controller in every sample.

Reasons for delays and jitter

- Computation time (possibly varying)
- Preemption (blocking) by other activities that are more important (have higher priority)
- Blocking due to access of shared resources
- Temporally non-deterministic implementation platform (hardware, OS)
- Communication delays

Outline

- Mechatronic Control Problems
- Discrete Control Systems
- Continuous Control Systems
  - Design Methodologies
  - Aliasing (Covered Thursday 27 Oct)
  - Arithmetics (Covered Thursday 27 Oct)
  - Computational Delay
  - **Implementation Paradigms**
- Example: PID Controller

Implementation Paradigms

Concurrent (parallel) activities.

A control system normally contains several more or less independent periodic or aperiodic activities/tasks (e.g., controllers)

It is often natural to handle the different tasks independently during design.

Temperature Loop

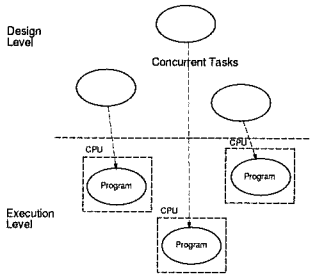
```
while (true) {
  Measure temperature;
  Calculate temperature error;
  Calculate the heater signal with PI–control;
  Output the heater signal;
  Wait for h seconds;
}
```

Level Loop

```
while (true) {
  Wait until level below L0;
  Open inlet valve;
  Wait unitil level above L1;
  Close inlet valve;
}
```


Paradigms

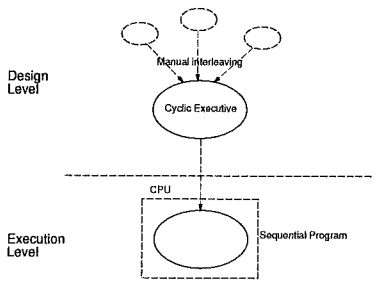
Parallel programming:



Multiprocessors, VLSI (ASIC), FPGA

Paradigms

Sequential programming:



Small micro-controllers.

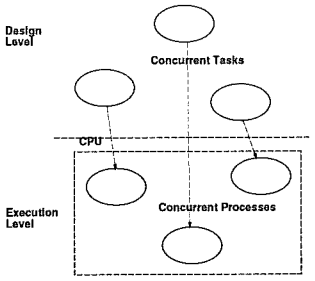
Interleaved temperature and level loops

```
while (true) {
  while (level above L0) {
    Measure temperature;
    Calculate temperature error;
    Calculate the heater signal with PI–control;
    Output the heater signal;
    Wait for h seconds;
  }
  Open inlet valve;
  while (level below L1) {
    Measure temperature;
    Calculate temperature error;
    Calculate the heater signal with PI–control;
    Output the heater signal;
    Wait for h seconds;
  }
  Close inlet valve;
}
```

Complex and non user-friendly code if programmed manually.

Paradigms

Concurrent programming:



The CPU is shared between the process (switches)

Real-Time Operating Systems or Real-time Programming Language with run-time system:

- switches between processes/threads
  - Real-Time Kernel
- timing primitives
- process communication

Lectures by Klas Nilsson.

Implementing Periodic Controller Tasks

Three Main Issues:

1. How do we achieve periodic execution?
2. When is the sampling performed?
3. When is the control signal sent out?

1. How do we achieve periodic execution?

Several options:

1. Using a static schedule (cyclic executive)?
   - High temporal determinism but inflexible
   - Does not require any sophisticated RTOS support
2. In interrupt handlers (interrupt service routines) associated with timers (typically in small microcontrollers)
3. As self-scheduling threads in a RTOS/kernel using time primitives such as sleep/delay/WaitTime (relative wait) or sleepUntil/delayUntil/WaitUntil (absolute wait)
4. Using an RTOS/kernel with built-in support for periodic tasks
   - implement the tasks as simple procedures/methods that are registered with the kernel
   - not yet common in commercial RTOS

Implementing Self-Scheduling Periodic Tasks

Attempt 1:

```
LOOP
  PeriodicActivity;
  WaitTime(h);
END;
```

Does not work.

Period $> h$ and time-varying.

The execution time of PeriodicActivity is not accounted for.

Implementing Self-Scheduling Periodic Tasks

Attempt 2:

```
LOOP
  Start = CurrentTime();
  PeriodicActivity;
  Stop = CurrentTime();
  C := Stop - Start;
  WaitTime(h - C);
END;
```

Does not work. An interrupt causing suspension may occur between the assignment and WaitTime.

In general, a WaitTime (Delay) primitive is not enough to implement periodic processes correctly.

A WaitUntil (DelayUntil) primitive is needed.

Implementing Self-Scheduling Periodic Tasks

Attempt 4:

```
LOOP
  t = CurrentTime();
  PeriodicActivity;
  t = t + h;
  WaitUntil(t);
END;
```

Does not work. An interrupt may occur between the WaitUntil and CurrentTime.

Implementing Self-Scheduling Periodic Tasks

Attempt 4:

```
t = CurrentTime();
LOOP
  PeriodicActivity;
  t = t + h;
  WaitUntil(t);
END;
```

Will try to catch up if the actual execution time of PeriodicActivity occasionally becomes larger than the period (a too long period is followed by a shorter one to make the average correct)

2. When is the sampling performed?

Two options:

- At the beginning of the controller task
  - gives rise to sampling jitter and, hence, sampling interval jitter
  - still quite common
- At the nominal task release instants
  - using a dedicated high-priority sampling task or in the clock interrupt handler
  - somewhat more involved scheme
  - minimizes the sampling jitter

3. When is the control signal sent out?

Three Options:

- At the end of the controller task
  - creates a longer than necessary input-output latency
- As soon as it can be sent out
  - minimizes the input-output latency
  - controller task split up in two parts: CalculateOutput and UpdateState
- At the next sampling instant
  - minimizes the latency jitter
  - gives a longer latency than necessary
  - often gives worse performance, also if the constant delay is compensated for
  - delay compensation easy

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An Example: PID Control

Textbook Algorithm:

$$u(t) = K(e(t) + \frac{1}{T_I}\int^t e(\tau)d\tau + T_D\frac{de(t)}{dt})$$

$$U(s) = K(E(s) + \frac{1}{sT_I}E(s) + T_D sE(s))$$

$$= P + I + D$$

A better algorithm

$$U(s) = K(\beta y_r - y + \frac{1}{sT_I}E(s) - \frac{T_D s}{1+sT_D/N}Y(s))$$

Modifications:

- Setpoint weighting (β) in the proportional term improves set-point response
- Limitation of the derivative gain (low-pass filter) to avoid derivation of measurement noise
- Derivative action only on y to avoid bumps for step changes in the reference signal

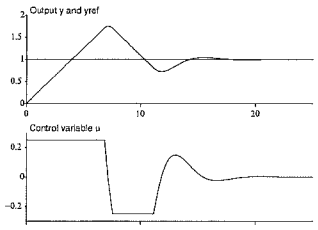
Control Signal Limitations

All actuators saturate.

Problems for controllers with integration.

When the control signal saturates the integral part will continue to grow – integrator (reset) windup.

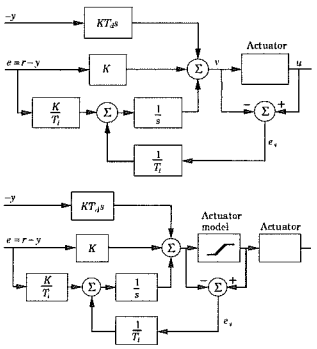
When the control signal saturates the integral part will integrate up to a very large value. This may cause large overshoots.



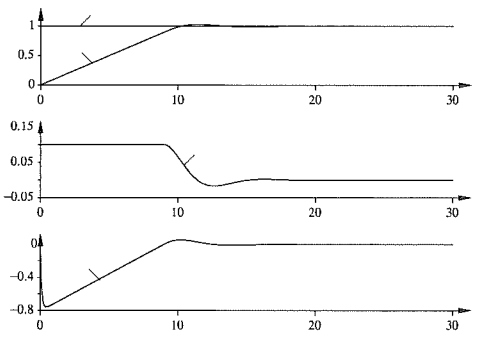
Tracking

- when the control signal saturates, the integral is recomputed so that its new value gives a control signal at the saturation limit
- to avoid resetting the integral due to, e.g., measurement noise, the re-computation is done dynamically, i.e., through a LP-filter with a time constant T_r.

Tracking



Tracking



Discretization

P-part:

$$u_P(k) = K(\beta y_{sp}(k) - y(k))$$

Discretization

I-part:

$$I(t) = \frac{K}{T_I}\int_0^t e(\tau)d\tau$$

$$\frac{dI}{dt} = \frac{K}{T_I}e$$

- Forward difference

$$\frac{I(t_{k+1}) - I(t_k)}{h} = \frac{K}{T_I}e(t_k)$$

```
I(k+1) := I(k) + (K*h/Ti)*e(k)
```

The I-part can be precalculated in UpdateStates

- Backward difference

The I-part cannot be precalculated, i(k) = f(e(k))

- Others

Discretization

D-part (assume $\gamma = 0$):

$$D = K\frac{sT_D}{1 + sT_D/N}(-Y(s))$$

$$\frac{T_D}{N}\frac{dD}{dt} + D = -KT_D\frac{dy}{dt}$$

- Forward difference (unstable for small T_D)
- Backward difference

$$\frac{T_D}{N}\frac{D(t_k) - D(t_{k-1})}{h} + D(t_k)$$

$$= -KT_D\frac{y(t_k) - y(t_{k-1})}{h}$$

$$D(t_k) = \frac{T_D}{T_D + Nh}D(t_{k-1})$$

$$-\frac{KT_DN}{T_D + Nh}(y(t_k) - y(t_{k-1}))$$

Discretization

Tracking:

```
v := P + I + D;
u := sat(v,umax,umin);
I := I + (K*h/Ti)*e + (h/Tr)*(u - v);
```


PID code

PID-controller with anti-reset windup

```
y = yIn.get(); // A-D conversion
e = yref - y;
D = ad * D - bd * (y - yold);
v = K*(beta*yref - y) + I + D;
u = sat(v,umax,umin)}
uOut.put(u);   // D-A conversion
I = I + (K*h/Ti)*e + (h/Tr)*(u - v);
yold = y
```

ad and bd are precalculated parameters given by the backward difference approximation of the D-term.

Execution time for CalculateOutput can be minimized even further.